UNCLASSIFIED

AD 228 384

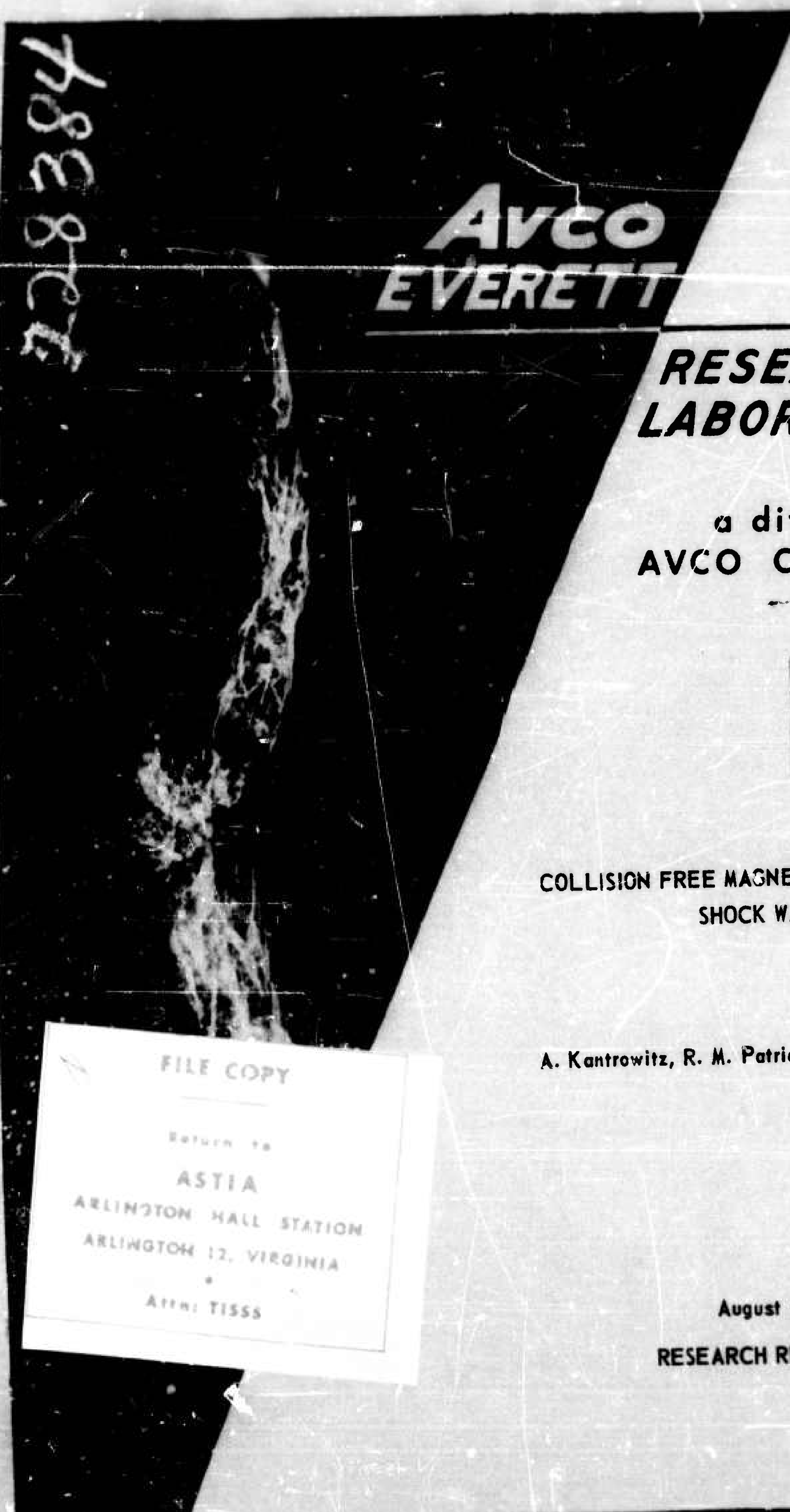
FOR MICRO-CARD CONTROL ONLY

1 OF 1

UNCLASSIFIED

228384

10



Avco Everett

RESEARCH LABORATORY

a division of
AVCO CORPORATION

COLLISION FREE MAGNETOHYDRODYNAMIC SHOCK WAVE

A. Kantrowitz, R. M. Patrick and H. E. Petschek

August 1959
RESEARCH REPORT 63

RESEARCH REPORT 63

COLLISION FREE MAGNETOHYDRODYNAMIC SHOCK WAVE

A. Kantrowitz, R. M. Patrick, and H. E. Petschek

AVCO-EVERETT
RESEARCH LABORATORY

Everett, Massachusetts

August 1959

ABSTRACT

It is assumed that the dissipation in a collision free shock produces a random distribution of magnetohydrodynamic waves. These waves are then treated as the fundamental particles of the plasma. A rough kinetic theory is developed which estimates the heat conduction coefficient due to the waves. Using this heat conduction coefficient, the shock thickness is estimated to be about 4 ~~four~~ times the characteristic ion Larmour radius. This prediction is in rough agreement with experimental results obtained in a MAST device.

COLLISION FREE MAGNETOHYDRODYNAMIC SHOCK WAVE*

by

A. Kantrowitz, R. M. Patrick and H. E. Petschek
Avco-Everett Research Laboratory
Everett, Massachusetts

Introduction

Collisional dissipation in plasma becomes very slow at high temperatures. It seems likely that other dissipative mechanisms associated with what has loosely been called "magnetohydrodynamic (MHD) turbulence" will then become of prime importance. A shock wave propagating perpendicular to the magnetic field in a plasma where the ion cyclotron radius is much smaller than the mean free path provides a good opportunity to study these phenomena. This paper reports progress on theoretical and experimental studies of these shock waves.

Wave Steepening and Thickest Shock Hypothesis

In many astrophysical or laboratory cases, shock waves are formed by the steepening of gradual compression fronts. As the steepness of the front increases diffusion processes become important and at some steepness can transfer sufficient momentum and energy and produce sufficient entropy so that a steady state shock profile is attained. One would therefore expect that the diffusion process which can act at the minimum steepness, i.e. at the longest range, will be the one which controls the shock structure provided that it can produce sufficient dissipation. This expectation will be referred to as the

---

* Sponsored jointly by the United States Air Force, Office of Scientific Research and Office of Naval Research under contracts AF 49(638)-61 and Nonr-2524(00) respectively.

thickest shock hypothesis.+

It has been suggested by Kahn[3] and by Parker[4] that a dissipative mechanism can be found in terms of plasma oscillations which leads to a shock thickness of the order of the Debye length. Gardner et al[5] have suggested that a permanent shock structure can be found from a series of pulses whose dimensions are of the order of the characteristic electron gyro-radius $\sqrt{\frac{mc^2}{4\pi Ne^2}}$ where m is the electron mass; N is the electron density; e is the electron charge; and c the velocity of light. In this paper we will attempt to show that in the presence of a transverse magnetic field, randomized MHD waves can provide the dissipation necessary for a steady state shock structure based on a length $r_i = \sqrt{\frac{Mc^2}{4\pi Ne^2}}$ the characteristic ion gyro-radius (M is the ion mass). The dissipative mechanism discussed here can provide the required dissipation for shocks whose velocity is less than about five times the upstream Alfven velocity ($M_A < 5$). For non-relativistic plasmas this length is orders of magnitude larger than the Debye length or the electron gyro radius

+ Several examples of the operation of the thickest shock hypothesis in collisional shock waves can be cited. For weak shock waves in CO_2 the dissipative mechanism is provided by the lag of vibrational energy behind the changes in translational energy. Steady shock structures have been observed experimentally which are of the order of 10^4 times thicker than the mean free path.[1] Similarly, in magnetohydrodynamic collisional shocks, Marshall[2] has shown that the Joule dissipation can provide the necessary transport and establish a thick shock wave unless the shock strength is too large.

and thus is preferred according to the thickest shock hypothesis.

<u>Proposed Collision Free Shock Model</u>

The model we propose is based on three facts:

1. MHD modes exist whose group velocity is faster than the propagation velocity of shock waves ($M_A \lessdot 5$). Thus, these waves can transfer energy upstream inside the shock front. Considering small amplitude waves propagating in a homogeneous, cold, collision-free plasma and neglecting displacement currents, the dispersion relation is

$$0 = \left(\omega \frac{Mc}{H_z e}\right)^4 \left(1 + \frac{m}{M} r_i^2 k^2\right)^2 - \left(\omega \frac{Mc}{H_z e}\right)^2 r_i^2 k^2 \Big[1 + \cos^2\theta + r_i^2 k^2 \left(\cos^2\theta + \frac{m}{M} \sin^4\theta\right)\Big] + r_i^4 k^4 \cos^2\theta \tag{1}$$

where ω is the frequency and $\vec{k}$ the wave number of the waves, H_z is the uniform applied magnetic field, and θ is the angle between H_z and $\vec{k}$. From eq. (1) the energy flow perpendicular to the magnetic field (i.e. the x component of the group velocity V_{gx}) can be found. We shall be interested in propagation directions chosen to maximize V_{gx} for a given k. The maximized V_{gx} values (which generally result for k $\approx 45^\circ$ to the magnetic field) are shown in Fig. 1. It will be seen that energy can be transferred upstream at speeds up to about five times the Alfven velocity ($u_{shock} = 5 V_A$).

2. MHD waves superposed on a compression front will grow in energy by compression. A wave packet will exert a pressure of the order of magnitude of the wave energy per unit volume ($\rho\varepsilon$) on the surrounding medium. If the surrounding medium is undergoing a compression, the work done against the pressure exerted by the wave packet will appear as increased energy of the wave packet. In the case

of a one-dimensional compression front propagating in the x direction, this work is given by $p_{xx}\frac{du}{dx}$ per unit time where p_{xx} is the stress exerted by the wave packet in the x direction across a plane perpendicular to x. It can be seen that if a wave packet spends a time in the shock front equal to the particle time in the shock front, then the ratio of its energy across the shock is similar to that experienced by a gas undergoing a similar isentropic compression. If, however, a wave packet spends a long time in the shock front, its energy can grow indefinitely.

3. The scattering of waves on waves can provide the randomizing process which is necessary to increase entropy in the shock front. Waves capable of affecting the dynamics of a shock front will have sufficient amplitude so that non-linear effects will be important. Thus, in particular, appreciable changes in magnetic field strength and propagation velocity will occur. (However, in the interesting k range, $kr_i \sim 10$, the accompanying density fluctuations and the plasma kinetic energies are small.) In analogy to the kinetic theory of gases we can estimate a mean free path λ_w travelled by a wave packet before appreciable scattering is produced by the random field. A wave will be appreciably altered in amplitude and direction when its phase has been changed by unity (or its "optical path" changed by $1/k$) due to the disturbing waves. If we approximate Eq. (1) by neglecting the angular dependence of the dispersion relation and assuming $r_i k \cos\theta \gg 1$, it may be written as

$$\omega \simeq \frac{eH_z}{Mc} \frac{(r_i k)^2}{1 + \frac{m}{M} r_i^2 k^2} \tag{2}$$

At constant frequency the change in wave number is then related to the perturbing magnetic field ΔH by

$$\frac{\Delta k}{k} = \frac{\Delta H}{H}\left(\frac{\partial \ln k}{\partial \ln H}\right)_{\omega} \simeq -\frac{1}{2}\frac{\Delta H}{H}\left(1+\frac{m}{M}r_i^2k^2\right) \tag{3}$$

The perturbing field of the other waves will be coherent over a distance of the order of the reciprocal of the mean wave number, i.e. $1/k_m$. In going this distance the phase of the wave therefore changes by an amount $-\frac{1+\frac{m}{M}r_i^2k^2}{2}\frac{\Delta H\,k}{H\,k_m}$. The phase changes by a random walk process with steps of this amount each time the wave goes a distance $1/k_m$. The length λ_w required to obtain an r.m.s. phase change of unity (the mean free path) is therefore given by the number $\sqrt{\lambda_w k_m}$ of steps required.

$$\lambda_w = \frac{4}{\beta k\left(1+\frac{m}{M}r_i^2k^2\right)^2}\left(\frac{k_m}{k}\right)^2 \tag{4}$$

where $\beta = \left(\frac{\Delta H}{H}\right)^2$ is the ratio of the average wave energy to the field energy. In later developments we will set $k_m = k$ thus assuming that only waves closely grouped about a single value of k have appreciable energy.

We are now prepared to construct a rough model of a shock front. Fig. 2 is drawn for a shock propagating at twice the Alfven velocity into a cold plasma. Our present state of knowledge does not allow us to say anything about the shock profile. For purposes of illustration and estimation of the shock thickness it has been assumed that the flow velocity and the average magnetic field vary linearly with distance through the shock front. Since waves are the dissipative mechanism the randomized energy will appear as wave energy behind

the shock, i.e.

$$\mathcal{E}_2 + \frac{p_2}{\rho_2} + \frac{H_2^2}{4\pi\rho_2} + \frac{u_2^2}{2} = \frac{H_1^2}{4\pi\rho_1} + \frac{u_1^2}{2} . \tag{5}$$

Variations of the expression on the left evaluated inside the shock front will depend on the precise nature of dissipative energy and momentum transfers inside the shock front. At the present writing we cannot say much about how this quantity will vary through the shock front. We will assume by analogy with ordinary collisional shocks that it is constant through the shock front. With the further assumption that the pressure and energy density of the waves are equal, the conservation laws now permit us to calculate a required energy transport upstream, q, which is also plotted non-dimensionally in Fig. 2. Note that in the center of a shock with $M_A = 2$ the wave energy has an average upstream group velocity about equal to the undisturbed Alfven speed.

Estimate of the Shock Thickness

The physical picture given above will be used to obtain order of magnitude estimates of the dominant wave length generated in the shock front and of the shock thickness. The energy flow upstream will be a diffusion process from high to low wave energy regions.

Roughly one can imagine the scattering of wave packets to be similar to the collision of gas molecules. The gradient of the wave energy $\frac{d\rho\mathcal{E}}{dx}$ will establish an anisotropic distribution of these waves which will result in a net energy flow upstream. On the other hand, the scattering processes will tend to create isotropy.

In analogy to the kinetic theory of heat conduction we will assume that the upstream energy flow is

$$q = \lambda_w (v_{gx} - u) \frac{d\rho\mathcal{E}}{dx} . \tag{6}$$

The thickest shock hypothesis can now be employed to estimate the dominant wave number. Clearly for a given energy density gradient waves with a maximum $\lambda_k(V'_{gx} - u)$ will have the greatest diffusivity and thus will lead to the thickest shock wave. We consider the case $M_A = 2$ and evaluate all quantities (except r_i & V_A) at the center of the shock. From eq. (4) and the calculations of V_{gx} quoted above, the value of k for maximum diffusivity was found to be $r_i k = 9$ and the diffusivity at this wave number was $2.5 r_i V_A$. The conservation laws give the energy gradients $\frac{1}{\rho\mathcal{E}} \frac{d\rho\mathcal{E}}{dx} = \frac{2}{L}$. Thus, equating the energy flow required by the conservation law with that given eq. (6), yields a shock thickness $L = 4 r_i$.

Similar calculations for other Mach numbers yield

for $M = 1.5$ $\quad$ $L = 10\, r_i$

$M = 3$ $\quad$ $L = 1\, r_i$

It is hardly necessary to point out that these thickness calculations are only order of magnitude estimates. It is very easy to make equally plausible arguments for shock thickness varying a factor of 3 either way from the values given here.

Comparison with Shock Tube Experiments

Shocks propagating perpendicular to an initial magnetic field in hydrogen have been produced at this laboratory in an annular electromagnetically driven shock tube (MAST).[6] With MAST it is possible to calculate the strength of the driving field and thus from the conservation laws the expected shock velocity. The velocity of the shock thus calculated agrees very well with experimental measurements. Shock velocities up to about 56 centimeters per microsecond have been produced. The density ratio across the shock can also be calculated from the conservation

laws. The density behind the shock front can be found from bremstrahlung measurements in the visible region with a photomultiplier. Interpreting this bremstrahlung in the terms of the plasma density assumes (1) that the gas is fully ionized, (2) that none of the hot plasma is lost to the wall, (3) that no appreciable contamination is present. Making these assumptions, the denisties have been calculated from the visible radiation and good agreement with the conservation law values for the cases of magnetic field parallel $(\rho_2/\rho_1 = 4)$ and perpendicular $(\rho_2/\rho_1 = 2.2)$ to the shock propagation direction have been found.

Shocks have been produced in which the characteristic ion cyclotron radius is as much as 100 times smaller than the mean free path downstream of the shock wave. It has been found that the density (light intensity) jumps and thus presumably the shock waves are much thinner than the downstream mean free path. It is not clear that the light intensity rise is indeed the shock thickness since it might be expected that the shock is not accurately plane. The shock thickness measurements presented on Fig. 3 are thus rough upperbounds and as can be seen a considerable scatter has been obtained.

The order of magnitude shock thickness estimate previously made has also been plotted on Fig. 3 and is in fortuitously good agreement with the measured shock thickness. Much work remains to be done before this agreement can be taken seriously.

Acknowledgment

It is a pleasure to acknowledge the advice and help of Dr. Frank Fishman in the theoretical portion of this work

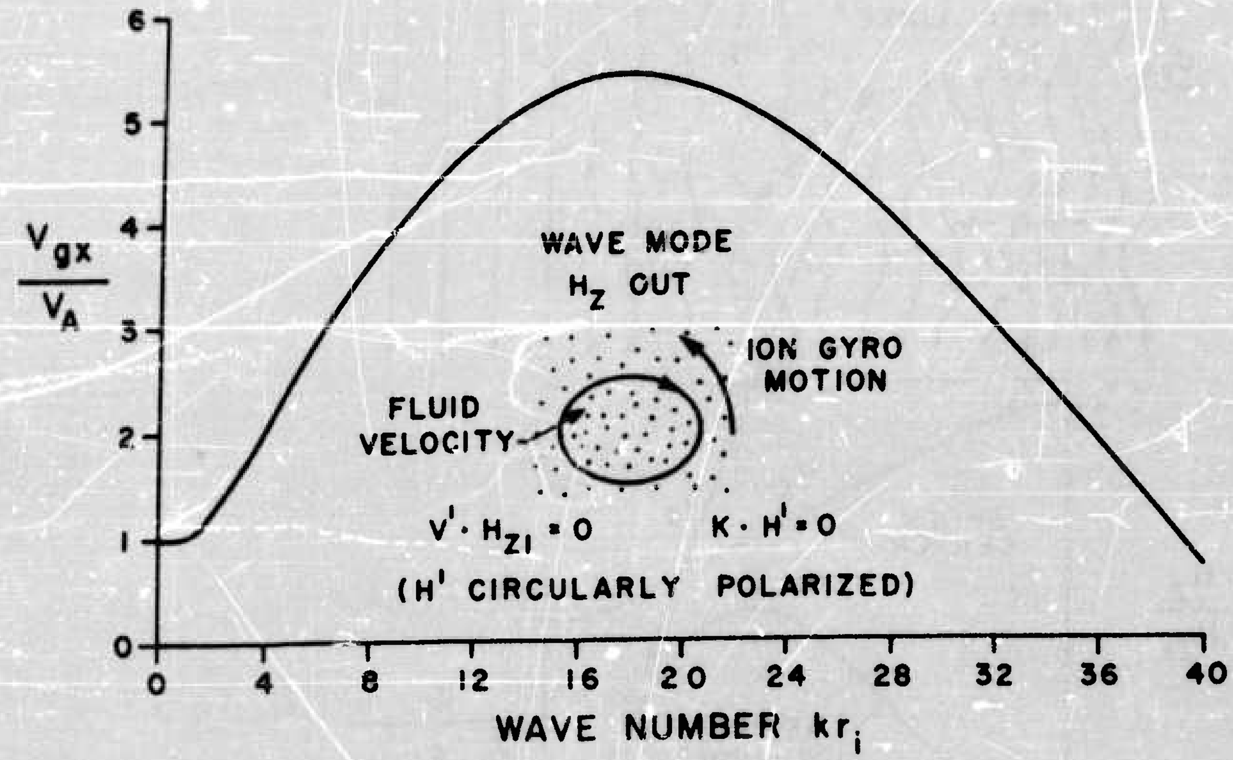


Fig. 1 The x component of the group velocity V_{gx} for "fast" MHD waves propagating in a cold plasma. The fast mode is circularly polarized in a direction opposite to the direction of rotation of the ions in the undisturbed magnetic field H_z. This mode is not heavily damped until frequencies close to the electron cyclotron frequency are reached. The plot is made for waves having propagation k chosen to maximize the V_{gx} for a given $|k|$.

3288

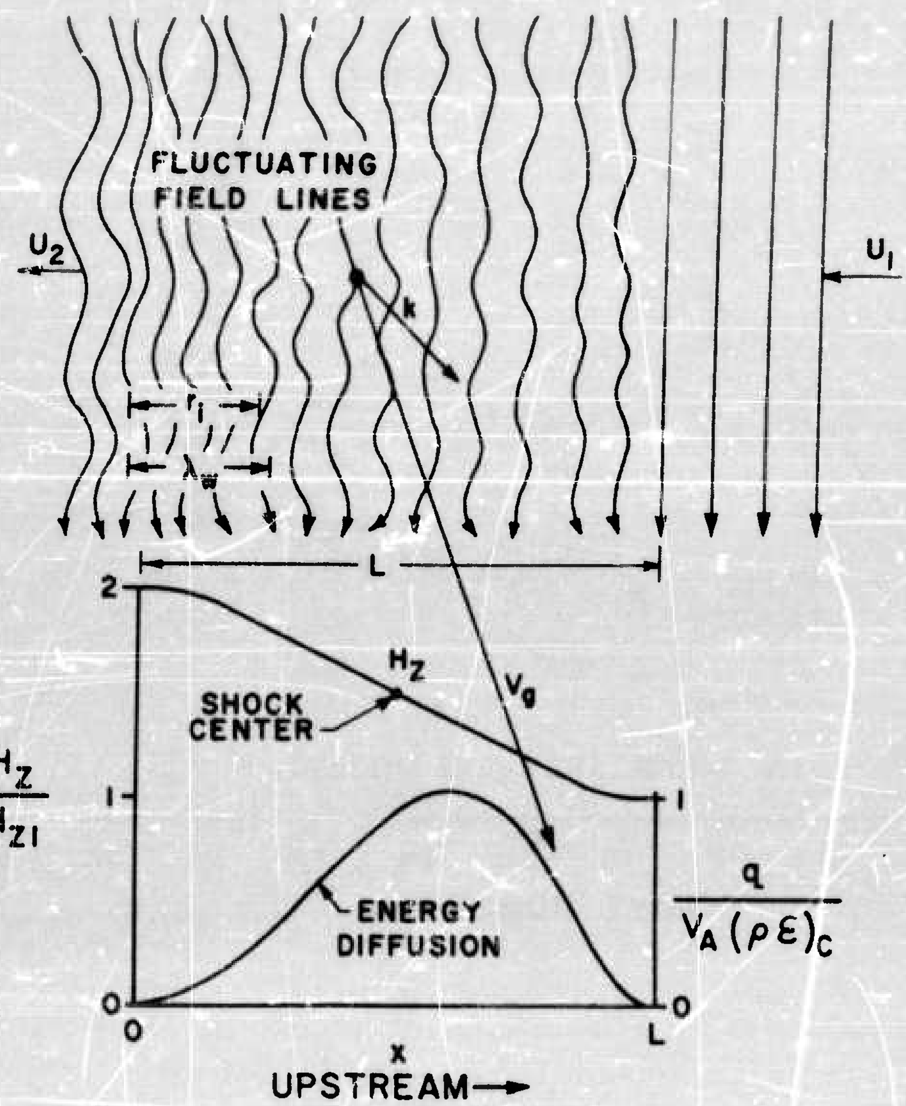


Fig. 2 Physical Picture of a Collision Free Shock Wave. The directed kinetic energy is converted in the shock front to randomized wave energy. The random wave amplitude required is estimated from the conservation laws. The conservation laws and the steady state condition also permit an estimate of the average upstream flux of energy and momentum which is also plotted.

3289

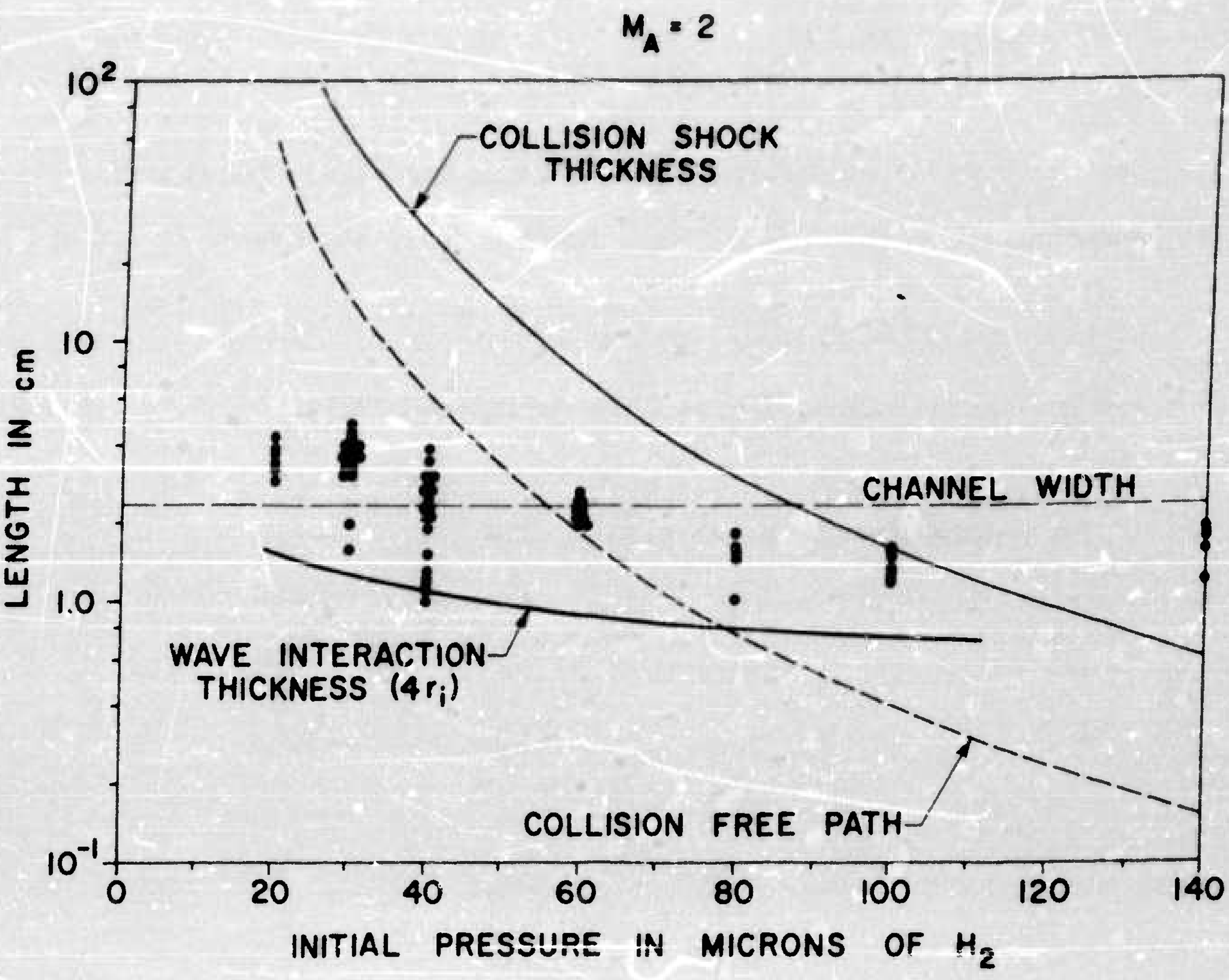


Fig. 3 Comparison of theoretical and experimental collision free shock thicknesses. Thickness measurements from bremstrahlung and theoretical estimates from the wave interaction model.

3286

REFERENCES

1. Griffith, W. C. and Kenny, Anne, J. Fluid Mech. 3, 286 (1957).

2. Marshall, W., Proc. Roy. Soc. London, 233, 367 (1955).

3. Kahn, F. D., Gas Dynamics of Cosmic Clouds, Chap. 20, p. 115-116, "The Collision of Two Highly Ionized Clouds."

4. Parker, E. N., Phys. Rev., to be published (1959).

5. Gardner, C. S., Goertzel, H., Grad, H., Morawetz, C. S., Rose, M. H., and Rubin, H. Proc. 2nd U. N. Int. Conf. on Peaceful Uses of Atomic Energy, Vol. 31, 15/P/374.

6. Patrick, R. M., "The Production and Study of High Speed Shock Waves in a Magnetic Annular Shock Tube," Avco-Everett Research Laboratory, Research Report 59, July 1959.